E C O N O M I C S   B U L L E T I N

# Rational expectations and ambiguity: A comment on Abel (2002)

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## *Abstract*

Abel (2002) proposes a resolution of the riskfree rate and the equity premium puzzles by considering pessimism and doubt. Pessimism is characterized by subjective probabilistic beliefs about consumption growth rates that are stochastically dominated by the objective distribution. The subjective distribution is characterized by doubt if it is a mean−preserving spread of the objective distribution. This note offers a decision theoretic foundation of Abel's ad−hoc definitions of pessimism and doubt under the assumption that individuals exhibit ambiguity attitudes in the sense of Schmeidler (1989). In particular, we show that the behavior of a representative agent, who resolves her uncertainty with respect to the true distribution of asset returns in a pessimistic way, is the equivalent to pessimism in Abel's sense. Furthermore, a representative agent, who takes into account pessimistic as well as optimistic considerations, may result in the equivalent to doubt in Abel's sense.

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# 1 Introduction

Abel (2002) convincingly argues that the assumption of *pessimism* and *doubt* may both help to resolve the *riskfree rate puzzle* (Weil, 1989) and the *equity premium puzzle* (Mehra and Prescott, 1985). By dropping the *rational expectations* assumption, Abel defines a *pessimist* as a decision maker whose subjective probabilistic belief about consumption growth rates is stochastically dominated by the objective distribution. Accordingly, a decision maker is characterized by *doubt* if her subjective probabilistic belief about consumption growth rates represents a mean-preserving spread of the objective distribution. As a shortcoming of his approach, Abel does not provide any further explanation why individuals might systematically commit such a specific violation of the *rational expectations* assumption.

The present note offers a decision-theoretic rationale for the occurrence of decision making that can be formally described as pessimism or doubt in the sense of Abel. Key to our approach is the assumption that individuals may exhibit *ambiguity attitudes* in the sense of Schmeidler (1989) and who may thus, for example, commit the Ellsberg Paradox (Ellsberg, 1961). Following Schmeidler, we formalize such individuals as CEU (=Choquet Expected Utility) decision makers, that is, they maximize expected utility with respect to *non-additive beliefs*. Properties of non-additive beliefs are used in the literature for formal definitions of, e.g., *ambiguity* and *uncertainty attitudes* (Schmeidler, 1989; Epstein, 1999; Ghirardato and Marinacchi, 2002), *pessimism* and *optimism* (Eichberger and Kelsey, 1999; Wakker, 2001; Chateauneuf et al., 2005), as well as *sensitivity to changes in likelihood* (Wakker, 2004).

Our approach focuses on non-additive beliefs that are defined as *neo-additive capacities* in the sense of Chateauneuf et al. (2005). Neo-additive capacities are non-additive beliefs that stand for marginal deviations from additive beliefs such that uncertainty is resolved by a combination of pessimistic and optimistic attitudes. In particular, a neo-additive capacity is characterized by a parameter $\delta$ (*degree of ambiguity*) which measures the lack of confidence the decision maker has in some additive probability distribution $\pi$. Moreover, the ambiguous part of a decision maker's belief puts some weight (measured by the *degree of optimism* $\lambda$) on the best consequence as well as some weight (measured by the *degree of pessimism* $\gamma = 1 - \lambda$) on the worst consequence possible.

In the context of Abel's model, we interpret this additive probability distribution $\pi$ as the representative agent's estimator for the underlying objective probability process of asset returns. Under the *rational expectations* paradigm the estimator $\pi$ must, first, coincide with the "true" probability distribution and, second, the individual must not be ambiguous about her subjective belief, i.e., $\delta = 0$. Analogously to the rational expectations approach, we assume that $\pi$ is indeed the correct estimator for the "true" probability distribution. However, our approach deviates from the rational expectations paradigm since we allow for the possibility that the decision maker is not entirely certain about whether her estimator $\pi$ coincides with the "true" probability distribution. Hence, $\delta > 0$ might be possible. The predominantly pessimistic (optimistic) CEU decision maker of our model then resolves this lack of confidence in her estimator $\pi$ in a pessimistic (optimistic) way by putting additional decision-weight on the possibility that the worst (best) consequence realizes for which $\gamma = 1$ ($\lambda = 1$).

Since the assumption of CEU decision makers with purely pessimistic beliefs successfully accommodates widely observed paradoxes of the Ellsberg type, our results support the presumption that real-life individuals can be formally described as pessimistic decision makers in the model of Abel. Even more relevantly, our decision theoretic foundation of Abel's assumption of doubt is related to recent empirical evidence showing that real-life decision makers take into account optimistic as well as pessimistic considerations (Kilka and Weber, 2001; Abdellaoui et al., 2004; Wakker, 2004).

We further demonstrate that the CEU of an act with respect to a neo-additive capacity can be equivalently described by the *$\alpha$-maxmin expected utility with respect to multiple priors* ($\alpha$-MEU) of an act which encompasses the original multiple priors approach of Gilboa and Schmeidler (1989) as a special case (see, e.g., Ghirardato et al., 1998; Ghirardato et al., 2004; Siniscalchi, 2005). In particular, we demonstrate the equivalence between the CEU with respect to neo-additive capacities and the $\alpha$-MEU with respect to so-called *$\varepsilon$-contaminated priors* used in Bayesian statistics (Berger and Berliner, 1986) that may be interpreted as neo-additive capacities.

Chen and Epstein (2002) also discuss Abel's ad hoc assumptions and base their model of ambiguity averse decision makers on the multiple priors model of Gilboa and Schmeidler (1989). While our motivation is similar, our approach differs in two important respects. First, while the maxmin expected utility model of Gilboa and Schmeidler (1989) can be used to equivalently describe pessimistic decision behavior in the sense of Abel, it cannot provide a formal equivalent for Abel's notion of doubt because - in contrast to the $\alpha$-MEU approach - it neglects any optimistic considerations in the case of ambiguity.[1] Second, our assumption of neo-additive capacities, where the subjective estimator $\pi$ just coincides with the true probability distribution represents only a slight - though in our opinion compelling - interpretational deviation from the rational expectations assumption. In our opinion, the concept of neo-additive capacities provides a more intuitive framework for modelling slight subjective distortions of true probabil-

[1] The concept of robust decision making as a deviation from expected utility theory due to cautious behavior recently developed in Hansen et al. (1999) and Anderson et al. (2003) does also not provide a formal equivalent for Abel's notion of doubt.

ity distributions than the multiple priors approach which is sometimes based on rather arbitrary sets of priors.

The remainder of this comment proceeds as follows. Section 2 introduces the reader to Choquet expected utility theory with a strong focus on neo-additive capacities. In Section 3 we demonstrate that a CEU decision maker with purely pessimistic beliefs can equivalently be formalized as a pessimist in the sense of Abel. We also show that CEU decision making which takes into account optimistic as well as pessimistic considerations is the analogue to doubt in Abel's sense. All formal proofs are relegated to the Appendix.

# 2 Choquet Expected Utility Theory and Neo-Additive Capacities

As a proposal for accommodating the Ellsberg paradox (Ellsberg, 1961), CEU theory was first axiomatized by Schmeidler (1986, 1989) for the framework of Anscombe and Aumann (1963) who assume the existence of random devices, generating objective probabilities. Subsequently, Gilboa (1987) as well as Sarin and Wakker (1992) have presented CEU axiomatizations for the Savage (1954) framework - where probabilities are derived from betting behavior as an exclusively personalistic concept - whereby Sarin and Wakker (1992) additionally assume the existence of *ambiguous* versus *unambiguous* events. CEU theory is equivalent to *cumulative prospect theory* (Tversky and Kahneman, 1992; Wakker and Tversky, 1993) restricted to the domain of gains (compare Tversky and Wakker, 1995). Moreover, as a representation of preferences over lotteries CEU theory coincides with *rank dependent utility theory* as introduced by Quiggin (1981, 1982), which is used to accommodate Allais-paradoxes (Allais, 1954).

Adopting the Anscombe-Aumann framework, we presume that the set of *consequences*, $X$, is some set of *lotteries* (=objective probability distributions). An *act*, $f$, is then a mapping from the set of states of the world into some set of *consequences*, i.e., $f : S \to X$. Given that preferences over acts satisfy the Schmeidler axioms, such preferences are representable by utility numbers that result from (Choquet-) integration of von Neumann-Morgenstern utility indices $u : X \to \mathbb{R}$ with respect to some *capacity.* A *capacity* (non-additive belief), $\nu$, on the state space $S$ is a real-valued set function on the subsets of $S$ which satisfies

(i) $\nu(\emptyset) = 0$, $\nu(S) = 1$

(ii) $A \subset B \Rightarrow \nu(A) \leq \nu(B)$

For $A \subset S$ let $u(f(A)) := u(f(s))$ if $u(f(s)) = u(f(s'))$ for all $s, s' \in A$. For a given act $f$ denote by $A_1, ..., A_m$ the partition of $S$ such that $u(f(A_1)) > ... > u(f(A_m))$.

Define

$$w(A_i) := [\nu(A_1 \cup ... \cup A_i) - \nu(A_1 \cup ... \cup A_{i-1})], \tag{1}$$

where we apply the convention that $\nu(A_1 \cup A_0) = 0$. Recall the definition of Choquet integration:

**Definition 1:** *The* Choquet expected utility *of an act $f$ with respect to capacity $\nu$ is defined by*

$$\mathbb{CEU}(f, \nu) := \sum_{i=1}^{m} u(f(A_i)) \cdot w(A_i) \tag{2}$$

**Definition 2** (Chateauneuf et al., 2005)**: Neo-additive capacities**

*A neo-additive capacity $\nu$ is defined as a linear combination of (i) an additive belief $\pi$, (ii) a non-additive belief $\omega^p$ (where only the universal event $S$ is considered as relevant), and (iii) a non-additive belief $\omega^o$ (where only the null event $\emptyset$ is considered as irrelevant). Formally:*

$$\nu(A) := (1-\delta) \cdot \pi(A) + \delta(\lambda \cdot \omega^o(A)) + \gamma \cdot \omega^p(A)))$$

*with $\delta \in (0,1]$, $\lambda, \gamma \in [0,1]$ such that $\lambda + \gamma = 1$, and*

$$\begin{aligned} \omega^o(A) &= 1 \text{ if } A \neq \emptyset \\ \omega^o(A) &= 0 \text{ if } A = \emptyset \end{aligned}$$

$$\begin{aligned} \omega^p(A) &= 0 \text{ if } A \subset S \\ \omega^p(A) &= 1 \text{ if } A = S \end{aligned}$$

The CEU of an act $f$ with respect to a neo-additive capacity $\nu$ is given by:

$$\begin{aligned} \mathbb{CEU}(f, \nu) &= (1-\delta) \cdot \sum_{i=1}^{m} \pi(A_i) \cdot u(f(A_i)) \\ &+\delta \cdot \left(\lambda \cdot \max_{s \in S} u(f(s)) + \gamma \cdot \min_{s \in S} u(f(s))\right). \end{aligned}$$

We refer to the parameter $\delta$ as the decision maker's *degree of ambiguity* since it has a straightforward interpretation as a measure of how confidently the individual believes that the additive measure $\pi$ indeed reflects the true probability distribution of an underlying random process. The individual's ambiguity about the additive measure $\pi$ is then resolved for neo-additive capacities by focussing on the extreme outcomes $\max_{s\in S} u(f(s))$ and $\min_{s\in S} u(f(s))$. How much an ambiguous individual cares about the best (worst) outcome possible for a chosen act is determined by her *degree of optimism* $\lambda \in [0,1]$ (*degree of pessimism* $\gamma \in [0,1]$). For example, if $\gamma = 1$ ($\lambda = 1$) we speak of a *purely pessimistic* (*optimistic*) decision maker since her ambiguity about the true probability leads her to particularly focus on the worst (best) consequence associated with her possible choices.

**Remark.** Notice that purely optimistic ($\lambda = 1$), respectively pessimistic ($\gamma = 1$), neo-additive capacities are *concave*, respectively *convex*, capacities. CEU decision makers with optimistic, respectively pessimistic, beliefs are therefore *ambiguity prone*, respectively *averse*, in the sense of Schmeidler's (1989) definition of ambiguity attitudes. As a consequence, CEU decision makers with purely pessimistic neo-additive capacities may commit the two-urn paradox as described in Ellsberg (1961), which violates the assumption that individuals actually decide under uncertainty as if they assigned some additive probability measure to events. More recent investigations (Kilka and Weber, 2004; Abdellaoui et al., 2004; Wakker, 2004) suggest that, besides expressing ambiguity aversion, most decision makers overweight the relevance of rather unlikely events so that a corresponding probability weighting function would be inversely S-shaped. Such a decision behavior can be well captured by CEU with respect to neo-additive capacities such that $0 < \gamma, \lambda$ and $\lambda \leq \gamma$.

# 3 A Decision Theoretic Foundation of Abel's Pessimism and Doubt

The representative individual of Abel's (2002) model (cf. also Lucas, 1978) holds some asset which produces returns $r$ according to some objectively given probability distribution $\pi$. Suppose that this asset may produce $m$ different returns, so that we can assume some finite partition $A_1, ..., A_m$ of the state space $S$ whereby greater indices of the events indicate lower returns, i.e., $r(A_j) > r(A_{j+1})$ for $j \in \{1, ..., m-1\}$.

In his proposal for a resolution of the *riskfree-rate* and the *equity premium puzzles*, Abel exploits the difference between the expected utility of the asset-returns with respect to the objective probability distribution, $\sum_{i=1}^{k} u(A_i) \cdot \pi(A_i)$, and the according expected utility of the asset-returns with respect to some subjective probability distribution $\pi^*$, i.e., $\sum_{i=1}^{k} u(A_i) \cdot \pi^*(A_i)$. Abel defines a pessimist as follows:

**Definition 3** (Abel, 2002)**:** *A decision maker is a* pessimist in the sense of Abel*, if and only if, her subjective probability distribution* $\pi^*$ *over asset-returns is (strictly) first-order stochastically dominated by the objective probability distribution* $\pi$*, i.e., for all* $k \in \{1, ..., m\}$*,*

$$\sum_{i=1}^{k} \pi^* (A_i) \leq \sum_{i=1}^{k} \pi (A_i)$$

*and for some* $k \in \{1, ..., m\}$*,*

$$\sum_{i=1}^{k} \pi^* (A_i) < \sum_{i=1}^{k} \pi (A_i)$$

Doubt in the sense of Abel is defined as follows:

**Definition 4** (Abel, 2002): *A decision maker is an* individual with doubt in the sense of Abel*, if and only if, her subjective probability distribution* $\pi^*$ *over asset-returns represents a mean-preserving spread of the objective probability distribution* $\pi$ *,i.e.,*

$$\mathbb{E}^*(r) = \sum_{i=1}^{k} \pi^* (A_i) \cdot r(A_i) = \sum_{i=1}^{m} \pi (A_i) \cdot r(A_i) = \mathbb{E}(r)$$

*and*

$$\text{var}^*(r) = \mathbb{E}^*(r - \mathbb{E}^*(r))^2 > \mathbb{E}(r - \mathbb{E}(r))^2 = \text{var}(r)$$

Observe that the only relevant act in Abel's model is *holding the asset*, so that a CEU decision maker with non-additive belief $\nu$ evaluates the asset as $\sum_{i=1}^{k} u (A_i) \cdot w (A_i)$ where $w (A_i)$ is given by (1).

We now show that our definition of a purely pessimistic CEU decision maker can be considered as a formal special case of Abel's definition.

**Proposition 1:** *A representative agent CEU decision maker with neo-additive capacity* $\nu$ *such that* $\gamma = 1$ *can be equivalently characterized as a pessimist in the sense of Abel whereby the subjective probability distribution* $\pi^*$ *is defined as follows:*

$$\pi_i^* := \begin{cases} (1 - \delta) \cdot \pi (A_i) & \text{for } i \in \{2, ..., m - 1\} \\ (1 - \delta) \cdot \pi (A_i) + \delta & \text{for } i = m. \end{cases}$$

We next demonstrate that a CEU decision maker might evaluate the asset in Abel's model **as if** she was an expected utility maximizer with subjective (additive) belief $\pi^*$ where $\pi^*$ is a mean-preserving spread of the true distribution $\pi$.

**Proposition 2:** *Consider a representative agent CEU decision maker with neo-additive capacity $\nu$ such that*

$$\mathbb{E}(r) = \lambda \cdot r(A_1) + \gamma \cdot r(A_m) \tag{3}$$

*and $\pi(A_1) + \pi(A_m) < 1$. Such a CEU decision maker can be equivalently characterized as an individual with doubt in the sense of Abel whereby the subjective probability distribution $\pi^*$ is defined as follows:*

$$\pi_i^* := \begin{cases} (1-\delta) \cdot \pi(A_i) & \text{for } i \in \{2, ..., m-1\} \\ (1-\delta) \cdot \pi(A_i) + \delta \cdot \lambda & \text{for } i = 1 \\ (1-\delta) \cdot \pi(A_i) + \delta \cdot \gamma & \text{for } i = m. \end{cases}$$

**Remark.** If the distribution of returns is symmetric, then assumption (3) holds iff $\lambda = \gamma = 0.5$, since, under symmetry, $r(A_1) - \mathbb{E}(r) = \mathbb{E}(r) - r(A_m)$.

**Remark.** The above results are established under the assumption that the CEU decision maker is the representative agent of the economy. An alternative way to read our results in Proposition 2 is to assume an economy that is populated by a proportion $\lambda$ of purely optimistic decision makers and a proportion $\gamma = 1 - \lambda$ of purely pessimistic decision makers.

**Remark.** As we show in the Appendix, the CEU of an act with respect to a neo-additive capacity can be equivalently described by the *$\alpha$-minmax expected utility with respect to multiple priors* ($\alpha$-MEU) (see, e.g., Ghirardato et al., 1998; Ghirardato et al., 2004; Siniscalchi, 2005). In particular, we establish this equivalence for so-called *$\varepsilon$-contaminated priors* used in Bayesian statistics (Berger and Berliner, 1986) under the two assumptions that (i) the "amount of error", $\varepsilon$, coincides with the degree of ambiguity, $\delta$, and (ii) the true probability distribution is contaminated with probability measures over the worst and best consequences.

# Appendix

## A.1 Formal Proofs

**Proof of Proposition 1:** Notice that (1) implies for purely pessimistic beliefs, i.e., $\gamma = 1$,

$$w_i = \begin{cases} (1-\delta) \cdot \pi(A_i) & \text{for } i \in \{1, ..., m-1\} \\ (1-\delta) \cdot \pi(A_i) + \delta & \text{for } i = m. \end{cases}$$

The last equation can be equivalently written as

$$w_m = 1 - (1-\delta) \cdot \sum_{i=1}^{m-1} \pi(A_i)$$

Now define $\pi_i^* := w_i$ for $i \in \{1, ..., m\}$, so that a CEU decision maker with neo-additive capacity $\nu$ evaluates the asset **as if** she was an expected utility maximizer with subjective (additive) belief $\pi^*$. Moreover, observe that

$$\sum_{i=1}^{k} \pi^*(A_i) = (1-\delta) \cdot \sum_{i=1}^{k} \pi(A_i) < \sum_{i=1}^{k} \pi(A_i) \text{ for } k \in \{1, ..., m-1\}$$

and

$$\sum_{i=1}^{m} \pi^*(A_i) = \sum_{i=1}^{m} \pi(A_i) = 1$$

Thus, the accordingly defined subjective pessimistic probability distribution $\pi^*$ is (strictly) first-order stochastically dominated by the objective probability distribution $\pi$. This proves our claim.□

**Proof of Proposition 2:** At first notice that assumption (3) entails

$$\begin{aligned} \mathbb{E}^*(r) &= \sum_{i=1}^{k} (1-\delta) \cdot \pi(A_i) \cdot r(A_i) + \delta (\lambda \cdot r(A_1) + \gamma \cdot r(A_m)) \\ &= (1-\delta) \cdot \mathbb{E}(r) + \delta \cdot \mathbb{E}(r) = \mathbb{E}(r), \end{aligned}$$

i.e., $\pi^*$ and $\pi$ have identical mean. Now turn to the variances:

$$\begin{aligned} \text{var}^*(r) &= \sum_{i=1}^{m} (1-\delta) \cdot \pi(A_i) \cdot [r(A_i) - \mathbb{E}(r)]^2 \\ &\quad + \delta \cdot \lambda \cdot [r(A_1) - \mathbb{E}(r)]^2 + \delta \cdot \gamma \cdot [r(A_m) - \mathbb{E}(r)]^2 \\ &= (1-\delta) \cdot \text{var}(r) + \delta \left(\lambda \cdot [r(A_1) - \mathbb{E}(r)]^2 + \gamma \cdot [r(A_m) - \mathbb{E}(r)]^2\right) \qquad (4) \end{aligned}$$

Since

$$\begin{aligned} & \lambda \cdot [r(A_1) - \mathbb{E}(r)]^2 + \gamma \cdot [r(A_m) - \mathbb{E}(r)]^2 \\ > & \ \pi(A_1) \cdot [r(A_1) - \mathbb{E}(r)]^2 + ... + \pi(A_m) \cdot [r(A_m) - \mathbb{E}(r)]^2 \\ = & \ \mathrm{var}(r) \end{aligned}$$

whenever assumption (3) holds and $\pi(A_1) + \pi(A_m) < 1$, equation (4) gives the desired result

$$\mathrm{var}^*(r) > \mathrm{var}(r),$$

i.e., the subjective probability distribution $\pi^*$ is a mean-preserving spread of $\pi$.□

## A.2 The $\alpha$-MEU Approach and $\varepsilon$-Contaminated Priors

For a finite state space the $\alpha$-MEU of an act $f$ is given by

$$\mathbb{MEU}(f, \alpha, P) = \alpha \cdot \min_{p \in P} \sum_{i=1}^{m} p(A_i) \cdot u(f(A_i)) + (1 - \alpha) \cdot \max_{p \in P} \sum_{i=1}^{m} p(A_i) \cdot u(f(A_i)),$$

where $P$ denotes some convex and compact set of priors. $\alpha$ is the weight associated with the minmax expected utility as developed in the original multiple prior approach of Gilboa and Schmeidler (1989) whereas $(1 - \alpha)$ is the weight associated with the maxmax expected utility (Ghirardato et al., 1998). In what follows we characterize the CEU with respect to a neo-additive capacity as an $\alpha$-MEU with respect to so-called *$\varepsilon$-contaminated priors* that are used in Bayesian statistics.

Rewrite the Choquet expected utility of act $f$ with respect to the neo-additive capacity $\nu$ as follows

$$\begin{aligned} \mathbb{CEU}(f, \nu) &= (1 - \delta) \cdot \sum_{i=1}^{m} \pi(A_i) \cdot u(f(A_i)) + \delta \cdot \left( \lambda \cdot \max_{s \in S} u(f(s)) + \gamma \cdot \min_{s \in S} u(f(s)) \right) \\ &= \alpha \cdot \sum_{i=1}^{m} \pi^{**}(A_i) \cdot u(f(A_i)) + (1 - \alpha) \cdot \sum_{i=1}^{m} \pi^{*}(A_i) \cdot u(f(A_i)) \end{aligned}$$

with

$$\begin{aligned} \alpha &= \gamma \Leftrightarrow \lambda = 1 - \alpha \\ \pi_i^* &= \begin{cases} (1 - \delta) \cdot \pi(A_i) & \text{for } i \in \{2, ..., m\} \\ (1 - \delta) \cdot \pi(A_i) + \delta & \text{for } i = 1 \end{cases} \\ \pi_i^{**} &= \begin{cases} (1 - \delta) \cdot \pi(A_i) & \text{for } i \in \{1, ..., m-1\} \\ (1 - \delta) \cdot \pi(A_i) + \delta & \text{for } i = m. \end{cases} \end{aligned}$$

Thus, we can represent CEU of an act with respect to neo-additive capacities by the $\alpha$-MEU of an act, i.e.,

$$\mathbb{MEU}(f,\alpha,P)=\mathbb{CEU}(f,\nu), \tag{5}$$

whenever we have, for the set of priors, $P$, that

$$\begin{aligned}\pi^{*} &\in \arg\max_{p\in P}\sum_{i=1}^{m}p(A_i)\cdot u(f(A_i)) \text{ and}\\ \pi^{**} &\in \arg\min_{p\in P}\sum_{i=1}^{m}p(A_i)\cdot u(f(A_i)).\end{aligned}$$

For instance, consider the set of priors, $\hat{P}$, that contains all probability measures $p$ that are dominated - in the sense of first order stochastic dominance - by $\pi^*$ whereas all $p\in P$ dominate $\pi^{**}$. In that case, (5) is satisfied for every possible specification of the vNM utility indices.[2]

A particularly convenient $\alpha$-MEU characterization of CEU with respect to neo-additive capacities results from so-called *$\varepsilon$-contaminated priors* (see, e.g., Berger and Berliner, 1986). For two probability measures $p_0$ and $q$, an $\varepsilon$-contaminated prior is defined as a probability measure

$$p=(1-\varepsilon)\cdot p_0+\varepsilon\cdot q$$

where $\varepsilon$ is interpreted as the "amount of error" by which $p_0$ is "contaminated" by the distribution $q$. Fix $\varepsilon=\delta$ and consider the *$\varepsilon$-contamination class*, $P^*$, given by

$$P^*:=\{p:p=(1-\varepsilon)\cdot\pi+\varepsilon\cdot q,q\in Q\}$$

such that

$$Q:=\{q:q=(1-\beta)\cdot I_1+\beta\cdot I_m,\ \beta\in[0,1]\},$$

where $I_1$ ($I_m$) denotes the degenerate probability measure that realizes the best (worst) consequence with probability one. Observe that $\pi^*$ dominates all $p\in P^*$ whereas every $p\in P^*$ dominates $\pi^{**}$, implying that

$$\mathbb{MEU}(f,\alpha,P^*)=\mathbb{CEU}(f,\nu).$$

That is, the CEU of an act $f$ with respect to a neo-additive capacity $\nu$ can be equivalently characterized as the $\alpha$-MEU of the act $f$ with respect to the $\varepsilon$-contamination class $P^*$. In that case, $\alpha$ coincides with the degree of pessimism, $\gamma$; the amount of error, $\varepsilon$, coincides with the degree of ambiguity, $\delta$; and the true probability measure, $\pi$, is

[2] Also compare Section 3.2.2 in Chateauneuf et al. (2005) who consider a strict subset of $\hat{P}$.

contaminated by all probability measures containing the best or the worst consequence in their support. Moreover, the $\varepsilon$-contaminated priors in $P^*$ can be interpreted as neo-additive capacities such that $\varepsilon$ denotes the degree of ambiguity and $\beta \in [0,1]$ denotes the degree of pessimism.

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